International Journal of Research in Advent Technology, Vol.6, No.5, May 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Deformation of a Bounded Poroelastic Half-Space in the Presence of a Point Sink

Amit Kumar*, Kuldip Singh, Mukesh Kumar Sharma
Department of Mathematics, Guru Jambheshwar University of Science & Technology, Hisar, Haryana, India
*Corresponding Author Email: pahuja1981@gmail.com

Abstract- Based on Biot's three-dimensional consolidation theory of porous media, analytical solutions for a saturated confined aquifer subjected to a constant-flux point sink are presented. In this study, porous medium is assumed to be isotropic, homogeneous and compressible. Also, the point sink can be located at an arbitrary depth in the aquifer. The closed-form solutions of the displacements and excess pore water pressure for a saturated confined aquifer are obtained by Laplace and Finite Fourier Transformation technique using suitable boundary conditions. These types of solutions are applicable to practical problems of finite plane strain poroelasticity in wide range of disciplines.

Key Words: Confined Aquifer, Poroelasticity, Point Sink, Laplace and Finite Fourier Transforms

1. INTRODUCTION

In recent years, flow deformation coupling in porous media has been more and more drawing our interest. It has been extensively applied to various engineering fields, such as consolidation of soft soil foundation under loading, land subsidence due to subsurface removal, stability of slopes, nuclear waste disposal and biological soft tissue deformation etc.

The theory of flow-deformation coupling in porous media originated from the research of Biot (1941) on the three dimensional consolidation of saturated soft soils under loads. Since then, many studies concerning coupled models with applications to various fields have been carried out by many researchers Gibson (1974), Booker and Carter(1987a, b),Selvadurai (2007), Tarn and Lu (1991) and others. In their studies, different assumptions or approximations were specified, e.g. different constitutive relations, anisotropic or isotropic, homogeneous or heterogeneous, saturated or unsaturated, and different forms of the effective stress law. All the above studies were developed on the classical theory of Biot's consolidation. In the meantime, Bowen (1980, 1982) presented the incompressible and compressible porous media models with the mixture theory which proved reliable with Biot's consolidation theory. Consequently, in the present problem, the flow-deformation coupling of poroelastic media has been carried out by direct application of Biot's consolidation theory.

There were a small number of exact solutions to coupled flow and deformation within finite two-dimensional porous media caused by fluid pumping but the bulk of them are for infinite or semi-infinite regions. Biot (1956a, 1956b) developed an exact solution of the deformation problem of vertical displacement while the upper surface of a semi-infinite plane domain was subjected to consistent loading of a definite width. Furthermore, the analytical solutions of axially symmetric plane strain consolidation problems under surface loading were derived by McNamee and Gibson (1960b). Booker and Carter (1986a, 1986b, and 1987b) found the steady state and time dependent exact solutions of deformation and flow due to constant flux point sink rooted in a saturated poroelastic half-space. In these solutions, soil properties were assumed to isotropic, while the permeability was assumed isotropic or transversely isotropic. Chen (2005) derived the steady state analytical solutions in a multilayered poroelastic half-space and investigated the effects of three kinds of pumping patterns and three kinds of boundary conditions upon the settlement. However, in practical engineering, the thickness of soft soil layer or aquifer is usually limited, and even though formation thickness is assumed to be infinite, the usual treatment method is to divide it into many thin layers according to the relevant profile of the engineering geology. Therefore, it is reasonably imperative and significant to study the finite two-dimensional problems.

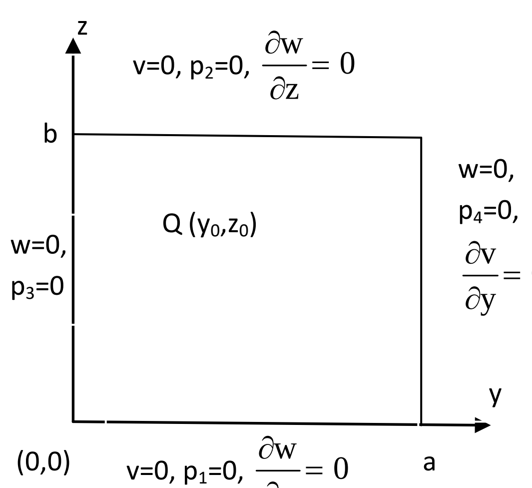
Bary and Mercer (1999) presented the analytical solution of displacement field and pressure field due to a point sink/source within a bounded two-dimensional incompressible porous media. They are applicable only for Dirichlet type boundary conditions of pore-pressure. Afterwards, the analytical solution of pore pressure field induced by a point sink with Neumann type boundary conditions within a bounded two-dimensional incompressible porous media were developed by Li and Lu

(2011). Accordingly, the focus of the present study is to obtain the analytical solution for the finite two-dimensional compressible poroelastic media based on the general Biot's consolidation theory.

In the present paper, we study the plane strain deformation of a waterlogged bounded aquifer subjected to a stable flux-point sink. The saturated aquifer is assumed to be homogeneous, isotropic, infiltrated by a single pore fluid following Darcy's law. The flow and deformation are regarded as quasi-static and the tensile stress is positive. The governing equations following Biot's consolidation theory have been directly applied to model coupled flow and deformation problem subjected to a point sink in the finite two-dimensional compressible aquifer. Both the displacement and pore-pressure fields in the physical domain are obtained using Laplace and Finite Fourier integral transforms and their inversions with suitable boundary conditions.

2. FORMULATION OF THE PROBLEM

The physical model of the problem is shown in figure 1 in which a saturated confined aquifer has been considered under the influence of a stable-flux point sink. The size of aquifer in x-direction is supposed to be much greater than the other two directions. As a consequence, strains associated with x-direction are much smaller than those in y-z cross section. That is, strains allied with x-direction can be ignored, and the problem under consideration reduces to the problem of plane-strain poroelasticity in y-z cross section.



The point sink is shown at an arbitrary depth at (y_0, z_0) .

Fig. 1. Physical Model of the problem

2.1 Governing Equations

As the problem under consideration is of plane-strain poroelasticity in y-z plane, setting $\sigma_{xx} = \sigma_{xy} = \sigma_{xz} = 0$ *and* $\frac{\partial}{\partial x} \equiv 0$, the constitutive behaviour of poroelastic model can be described with the help of following relations::

Hooke's law

$$\sigma_{yy} = 2G\varepsilon_{yy} + 2G\frac{\upsilon}{1-2\upsilon}\varepsilon_{kk} - \alpha\, p \qquad (1)$$

$$\sigma_{zz} = 2G\varepsilon_{zz} + 2G\frac{\upsilon}{1-2\upsilon}\varepsilon_{kk} - \alpha\, p \qquad (2)$$

$$\sigma_{yz} = 2G\varepsilon_{yz} \qquad (3)$$

Strain-Displacement Relations

The strains ε_{ij} are related to the displacements u_i in the following manner:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (4)$$

where i, j= 2, 3

The total stress must satisfy the following

Equilibrium Equations

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + F_y = 0 \qquad (5)$$

$$\frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + F_z = 0 \qquad (6)$$

Displacement-Pore pressure equation

If Hooke's law given by equations (1) - (3) and equations of equilibrium given by (5) - (6) are combined, in the absence of body forces, it is found that

$$G\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{G}{1-2\upsilon}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right) - \alpha\frac{\partial p}{\partial y} = 0 \qquad (7)$$

$$G\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{G}{1-2\upsilon}\left(\frac{\partial^2 v}{\partial z \partial y} + \frac{\partial^2 w}{\partial z^2}\right) - \alpha\frac{\partial p}{\partial z} = 0 \qquad (8)$$

The equation for the pore-pressure field is

$$\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=\frac{\alpha}{\lambda_f}\frac{\partial \varepsilon_v}{\partial t}+\frac{1}{\chi}\frac{\partial p}{\partial t}-\frac{1}{\lambda_f}Q=0 \tag{9}$$

In all the above equations from (1) - (9), the symbols v, w denote the solid displacement components in y and z coordinate directions respectively.

$G=\frac{E}{2(1+\upsilon)}$ is the shear modulus, E is the Young's modulus and υ is the Poisson's ratio. $\varepsilon_v=\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$ is the bulk volumetric strain.

$\alpha=1-\frac{K_b}{K_s}$ is the Biot's poroelastic co-efficient, $K_b=\frac{E}{3(1-2\upsilon)}$ is the bulk modulus of porous medium, K_s is the bulk modulus of solid grains, p is the pore pressure and Q is the source intensity per unit bulk volume(positive for source and negative for sink).

$\lambda_f=\frac{k_f}{\mu_f}$ denote the fluidity of pore fluid, k_f is the absolute permeability and μ_f the viscosity of pore fluid.

$\chi=\frac{\lambda_f}{\phi C_t}$ is the transmission coefficient, $\phi C_t=\frac{\phi}{K_f}+\frac{\alpha-\phi}{K_s}$ and C_t is known as bulk compressibility of pore fluid.

Re-arranging equations (7) and (8) and then dividing by G, we get

$$(n+1)\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}+n\frac{\partial^2 w}{\partial y\partial z}-\frac{\alpha}{G}\frac{\partial p}{\partial y}=0 \tag{10}$$

$$(n+1)\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial y^2}+n\frac{\partial^2 v}{\partial y\partial z}-\frac{\alpha}{G}\frac{\partial p}{\partial z}=0 \tag{11}$$

where $n=\frac{1}{1-2\nu}$

Accordingly, the governing equations for flow and deformation coupling within a bounded aquifer are given by Equations (9), (10) and (11).

2.2 Initial and Boundary Conditions

Displacement field boundary conditions $on\ z=0\ and\ z=b$ and $on\ y=0\ and\ y=a$ are given as follows:

$$\left.\begin{aligned} v=0,\quad \frac{\partial w}{\partial z}=0\quad on\ z=0\ and\ z=b\\ w=0,\quad \frac{\partial v}{\partial y}=0\quad on\ y=0\ and\ y=a\end{aligned}\right\} \tag{12}$$

Pore-pressure boundary conditions

$$\left.\begin{aligned} p=p_1(y,t)=0\quad on\ z=0\\ p=p_2(y,t)=0\quad on\ z=b\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} p=p_3(z,t)=0\quad on\ y=0\\ p=p_4(z,t)=0\quad on\ y=a\end{aligned}\right\} \tag{14}$$

It should be noted that the displacement field boundary conditions in the form (12) are particularly preferred which are to go with the following Finite Fourier Transformations and make simpler the formulation procedure.

The displacement and pore-pressure field initial conditions at $t=0$

$$\left.\begin{aligned} v(y,z)=w(y,z)=0\quad at\ t=0\\ p(y,z)=0\quad at\ t=0\end{aligned}\right\} \tag{15}$$

3. ANALYTICAL SOLUTIONS

The Laplace transform, the finite cosine transform and finite sine transform, respectively, of a function f are defined as follows:

$$\left.\begin{aligned} L\{f(t)\}&=\int_0^\infty f(t)e^{-st}dt\\ C_{ym}\{f(y)\}&=\int_0^a f(y)\cos(\gamma_m y)dy\\ C_{zr}\{f(z)\}&=\int_0^b f(z)\cos(\gamma_r z)dz\\ S_{ym}\{f(y)\}&=\int_0^a f(y)\sin(\gamma_m y)dy\\ S_{zr}\{f(z)\}&=\int_0^b f(z)\sin(\gamma_r z)dz\end{aligned}\right\} \tag{16}$$

where $\gamma_m=\frac{m\pi}{a},\quad \gamma_r=\frac{r\pi}{b}\qquad m,r=0,1,2,3,4........$

The governing partial differential equations (9), (10) and (11) can be reduced to the following differential equations by applying $S_{zr}\{Eq.9\}$, $S_{zr}\{Eq.10\}$, $C_{zr}\{Eq.11\}$ using boundary conditions given by (12) - (14).

$$\left.\begin{aligned}&\frac{d^2\tilde{p}_s}{dy^2}-\gamma_r^2\tilde{p}_s=\frac{\alpha}{\lambda_f}\frac{d}{dt}\left(\frac{d\tilde{v}_s}{dy}-\gamma_r\tilde{w}_c\right)+\frac{1}{\chi}\frac{d\tilde{p}_s}{dt}-\frac{\tilde{Q}_s}{\lambda_f}\\&(n+1)\frac{d^2\tilde{v}_s}{dy^2}-\gamma_r^2\tilde{v}_s-n\gamma_r\frac{d\tilde{w}_c}{dy}-\frac{\alpha}{G}\frac{d\tilde{p}_s}{dy}=0\\&\frac{d^2\tilde{w}_c}{dy^2}-\gamma_r^2(n+1)\tilde{w}_c+n\gamma_r\frac{d\tilde{v}_s}{dy}-\frac{\alpha}{G}\gamma_r\tilde{p}_s=0\end{aligned}\right\}\qquad(17)$$

where
$\tilde{p}_s(y,r,t)=S_{zr}\left(p(y,z,t)\right)$, $\tilde{v}_s(y,r,t)=S_{zr}\left(v(y,z,t)\right)$,
$\tilde{Q}_s(y,r,t)=S_{zr}\left(Q(y,z,t)\right)$ and $\tilde{w}_c(y,r,t)=C_{zr}\left(w(y,z,t)\right)$

Now applying S_{ym}, C_{ym} and S_{ym} on first, second and third equation, respectively, of (17), we obtain the following set of differential equations with the help of boundary conditions given by (12)-(14).

$$\left.\begin{aligned}&-(\gamma_m^2+\gamma_r^2)\,\hat{p}_{ss}+\frac{\alpha}{\lambda_f}\frac{d}{dt}\left(\gamma_m\hat{v}_{sc}+\gamma_r\hat{w}_{cs}\right)-\frac{1}{\chi}\frac{d\hat{p}_{ss}}{dt}=-\frac{\hat{Q}_{ss}}{\lambda_f}\\&(n+1)\gamma_m^2\hat{v}_{sc}+\gamma_r^2\hat{v}_{sc}+n\gamma_m\gamma_r\hat{w}_{cs}+\frac{\alpha}{G}\gamma_m\hat{p}_{ss}=0\\&(n+1)\gamma_r^2\hat{w}_{cs}+\gamma_m^2\hat{w}_{cs}+n\gamma_m\gamma_r\hat{v}_{sc}+\frac{\alpha}{G}\gamma_r\hat{p}_{ss}=0\end{aligned}\right\}\qquad(18)$$

where
$\hat{p}_{ss}(m,r,t)=S_{xm}S_{zr}\left(p(y,z,t)\right)$, $\hat{v}_{sc}(m,r,t)=C_{xm}S_{zr}\left(v(y,z,t)\right)$,
$\hat{Q}_{ss}(m,r,t)=S_{xm}S_{zr}\left(Q(y,z,t)\right)$ and $\hat{w}_{cs}(m,r,t)=S_{xm}C_{zr}\left(w(y,z,t)\right)$

Further applying Laplace transformation w.r.t. time variable t and using initial conditions given by (15), the differential equations in (18) are reduced to following algebraic equations:

$$\left.\begin{aligned}&\frac{\alpha}{\lambda_f}s\gamma_n\bar{v}+\frac{\alpha}{\lambda_f}s\gamma_q\bar{w}-(\tilde{\gamma}+\frac{s}{\chi})\bar{p}=-\frac{\bar{Q}}{\lambda_f}\\&n\gamma_m^2\bar{v}+\tilde{\gamma}\bar{v}+n\gamma_m\gamma_r\bar{w}+\frac{\alpha}{G}\gamma_m\bar{p}=0\\&n\gamma_r^2\bar{w}+\tilde{\gamma}\bar{w}+n\gamma_m\gamma_r\bar{v}+\frac{\alpha}{G}\gamma_r\bar{p}=0\end{aligned}\right\}\qquad(19)$$

where
$\bar{p}(m,r,s)=LS_{xm}S_{zr}\left(p(y,z,t)\right)$, $\bar{v}(m,r,s)=LC_{xm}S_{zr}\left(v(y,z,t)\right)$,
$\bar{Q}(m,r,s)=LS_{xm}S_{zr}\left(Q(y,z,t)\right)$ and $\bar{w}(m,r,s)=LS_{xm}C_{zr}\left(w(y,z,t)\right)$

$\tilde{\gamma}=\gamma_m^2+\gamma_r^2$ and s is the Laplace transform variable.

Solving the equations w.r.t. $\bar{v}(m,r,s),\bar{w}(m,r,s)$ and $\bar{p}(m,r,s)$, we obtain

$$\left.\begin{aligned}&\bar{v}(m,r,s)=\frac{-\bar{Q}\,\alpha\gamma_m}{\bar{\gamma}[\alpha^2 s+G(n+1)\lambda_f(\bar{\gamma}+s/\chi)]}\\&\bar{w}(m,r,s)=\frac{-\bar{Q}\,\alpha\gamma_r}{\bar{\gamma}[\alpha^2 s+G(n+1)\lambda_f(\bar{\gamma}+s/\chi)]}\\&\bar{p}(m,r,s)=\frac{\bar{Q}G(n+1)}{[\alpha^2 s+G(n+1)\lambda_f(\bar{\gamma}+s/\chi)]}\end{aligned}\right\}\qquad(20)$$

As we are considering the case of constant-flux point sink, therefore

$$Q=-Q_0\delta(y-y_0)(z-z_0)\qquad(21)$$

Applying $LS_{ym}S_{zr}$ on equation (21), we obtain

$$\bar{Q}=\frac{-Q_0\sin(\gamma_m y_0)\sin(\gamma_r z_0)}{s}\qquad(22)$$

Substituting equation (22) into (20) and taking Laplace inversion w.r.t. s yields

$$\left.\begin{aligned}&\hat{v}(m,r,t)=\frac{\alpha\gamma_m Q_0\sin(\gamma_m y_0)\sin(\gamma_r z_0)(1-e^{-\eta t})}{\tau}\\&\hat{w}(m,r,t)=\frac{\alpha\gamma_r Q_0\sin(\gamma_m y_0)\sin(\gamma_r z_0)(1-e^{-\eta t})}{\tau}\\&\hat{p}(m,r,t)=\frac{-Q_0\sin(\gamma_m y_0)\sin(\gamma_r z_0)(1-e^{-\eta t})}{\lambda_f\tilde{\gamma}}\end{aligned}\right\}\qquad(23)$$

where $\eta=\dfrac{G\lambda_f\tilde{\gamma}(n+1)}{\alpha^2+G\lambda_f(n+1)/\chi}$ and $\tau=G\lambda_f(n+1)\tilde{\gamma}^2$

Consequently, the poroelastic analytical solutions in the physical domain with a stable-flux point sink given by taking double inversion to equations (23) are as follows:

$$v(y,z,t)=\frac{2}{ab}\sum_{r=1}^{\infty}\hat{v}(0,r,t)\sin(\gamma_r z)+\frac{4}{ab}\sum_{r=1}^{\infty}\sum_{m=1}^{\infty}\hat{v}(m,r,t)\cos(\gamma_m y)\sin(\gamma_r z)\qquad(24)$$

$$w(y,z,t)=\frac{2}{ab}\sum_{m=1}^{\infty}\hat{w}(m,0,t)\sin(\gamma_m y)+\frac{4}{ab}\sum_{r=1}^{\infty}\sum_{m=1}^{\infty}\hat{w}(m,r,t)\sin(\gamma_m y)\cos(\gamma_r z)\qquad(25)$$

$$p(y,z,t)=\frac{4}{ab}\sum_{r=1}^{\infty}\sum_{m=1}^{\infty}\hat{p}(m,r,t)\sin(\gamma_m y)\sin(\gamma_r z)\qquad(26)$$

where $\hat{v}(m,r,t),\ \hat{w}(m,r,t)\ and\ \hat{p}(m,r,t)$ are given by equation (23).

4. RESULTS

For the sake of numerical calculations three non-dimensional quantities are defined as

$$w^* = \frac{w}{b},\ y^* = \frac{y}{a},\ t^* = \frac{\lambda_f (n+1)G}{b^2} t$$

Figure 2 shows the profile of non-dimensional vertical displacement w^* at the centre (y=0.5a, z=b) of the upper surface against the non-dimensional time t^*. It is observed that transient response of w^* at the centre (y=0.5a, z=b) exhibits the trends of exponential decaying. In addition to it, w^* tends to be steady when t^* becomes sufficiently large (around $t^* > 1.6$). This is due to the dissipation effect occurring in the consolidation process.

Figure 3 shows the long term distribution of non-dimensional vertical displacement w^* on the upper surface against the of non-dimensional horizontal coordinate y^* It is observed that the curve of w^* on the upper surface (z=b) versus y^* presents the trend of complete symmetric funnel type distribution with respect to $y^* = 0.5$. This is due to the central location of the point sink and the symmetry of boundary conditions.

5. DISCUSSION AND CONCLUSIONS

This paper presents an exact solution for the transient two-dimensional flow and deformation of saturated confined aquifer. The assumptions of an isotropic, homogeneous and compressible aquifer are taken into consideration in the present study. The general theory of Biot's consolidation has been used to govern the fluid-solid interaction. First type (Dirichlet) type boundary conditions of fluid flux and the corresponding suitable displacement field boundary conditions are considered. In the derivation, appropriate finite sine and cosine transforms and Laplace transforms are specifically chosen to simplify the transforms of the governing equations. The proposed analytical solution can help us obtain in-depth insights into time-dependent mechanical behaviour due to fluid extraction within finite two-dimensional porous media. Furthermore, it can also be of huge importance to calibrate numerical solutions in plane strain poroelasticity and to formulate appropriate industry norms and principles.

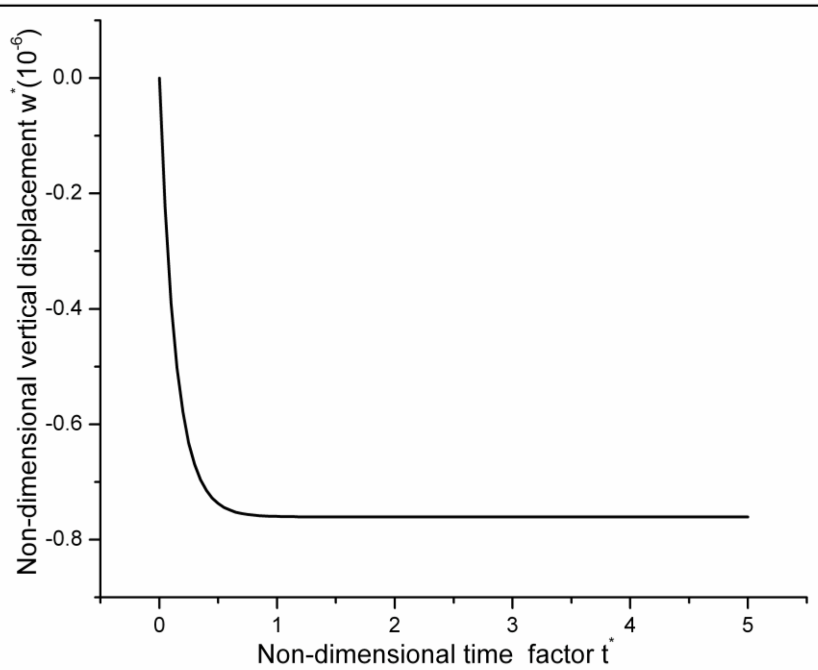


Fig. 2. The transient response of the vertical displacement w^* at the centre (y=0.5a, z=b) of the upper surface against the time factor t^*

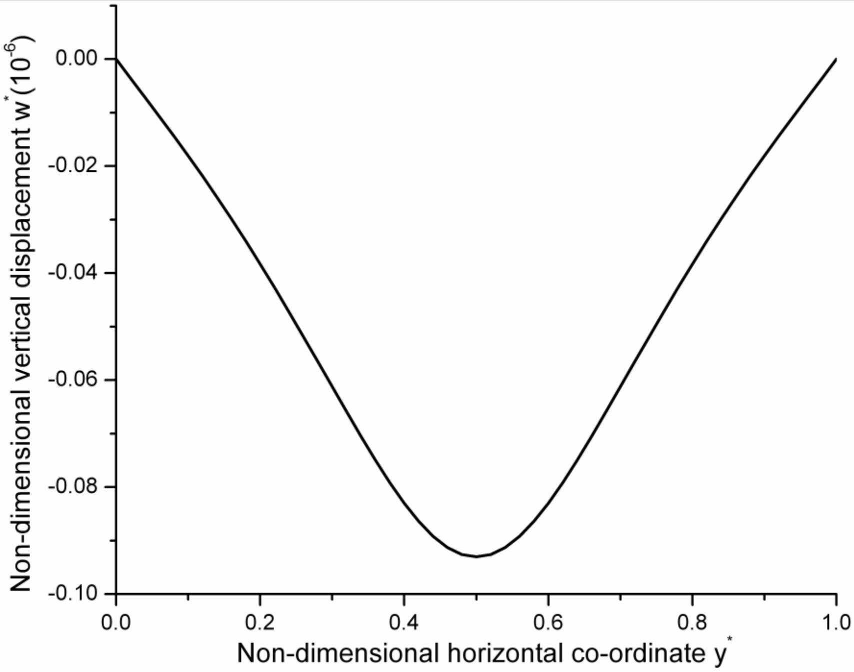


Fig. 3. The trend of long term distribution of w^* on the upper surface (z=b) against the dimensionless horizontal coordinate y^* is complete symmetric funnel type distribution with respect to y^*= 0.5

REFERENCES

[1] Biot, M.A. (1941). "General theory of three-dimensional consolidation." J.Appl. Phys., 12(2), 155-164.

[2] Booker, J.R., and Carter, J.P. (1986a). "Analysis of a point sink embedded in a porous elastic half-space." Int. J. Numer. Anal. Methods Geomech., 10(2), 137-150.

[3] Booker, J.R., and Carter, J.P. (1986b). "Long term subsidence due to fluid extraction from a saturated, anisotropic elastic soil mass." Q. J. Mech. Appl. Math., 39(1), 85-97.

[4] Booker, J.R., and Carter, J.P. (1987a). "Elastic consolidation around a point sink embedded in a half-space with anisotropic permeability." Int. J. Numer. Anal. Methods Geomech., 11(1), 61-77.

[5] Booker, J.R., and Carter, J.P. (1987b). "Withdrawal of a compressible pore fluid from a point sink in an isotropic elastic half-space with anisotropic permeability." Int. J. Solids Struct., 23((3), 369-385.

[6] Booker, J.R., and Savvidou, C. (1984). "Consolidation around a spherical heat source." Int. J. Solids Struct., 20(11), 1079-1090.

[7] Booker, J.R., and Savvidou, C. (1985). "Consolidation around a point heat source." Int. J. Numer. Anal. Methods Geomech., 9(2), 173-184.

[8] Bear, J. and Corapcioglu, M.Y., (1981a). "Centrifugal filtration in deformable porous media." In: Bear, J. and Corapcioglu, M.Y. (eds.), A Series of Four Papers on Water Flow in Deformable Porous Media. Technical Report UMR-0284, Department of Civil Engineering, University of Michigan, Ann Arbor, Michigan, Section I, 42 p.

[9] Bear, J. and Corapcioglu, M.Y., 1981b, Mathematical model for regional land subsidence due to pumping: 2. Integrated aquifer subsidence equations for vertical and horizontal displacements.Water Resources Research, 17, 947−958.

[10] Chen, G. J. (2004). "Consolidation of multilayered half space with anisotropic permeability and compressible constituents." *Int. J. Solids Struct.* 41, 4567–4586.

[11] Chen, G. J. (2005). "Steady-state solutions of multilayered and cross-anisotropic poroelastic half-space due to a point sink." *Int. J. Geomech.* 5**,** 45–57.

[12] Erdelyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F.G. (1954). Tables of integral transforms, McGraw-Hill, New York.

[13] Feng-Tsai Lin and John C.-C. Lu (2010). Golden Ratio in the Point Sink Induced Consolidation Settlement of a Poroelastic Half Space, Modeling Simulation and Optimization - Tolerance and Optimal Control, Shkelzen Cakaj (Ed.).

[14] Kim, J.M. (2004), "Fully coupled poroelastic governing equations for groundwater flow and solid skeleton deformation in variably saturated true anisotropic porous geologic media." Geosciences Journal, 8(3), 291—300.

[15] Love, A.E.H. (1944). A treatise on the Mathematical Theory of Elasticity, Dover press, New York.

[16] Poland, J.F. (1984). Guidebook to studies of land subsidence due to groundwater withdrawal, The United nations Educational Scientific and Cultural organization, Paris.

[17] Safai, N.M. and Pinder, G.F., (1979). " Vertical and horizontal land deformation in a desaturating porous medium." Advances in Water Resources, 2, 19–25.

[18] Savvidou, C. and Booker, J.R. (1989). "Consolidation around a heat source buried deep in a porous thermoelastic medium with anisotropic flow properties" Int. J. Numer. Anal. Methods Geomech., 13(1), 75-90.

[19] Singh, S.J., Rani, S. and Kumar, R. (2007). "Quasi-static deformation of a poroelastic half space with anisotropic permeability by two-dimensional surface loads." *Geophys. J. Int.,* 170, 1311–1327.

[20] Terzaghi, K. (1925). Erdbaumechanik auf Bodenphysikalischer Grundlage. Franz Deuticke, Leipzig und Wien, 399 p.

[21] Verruijt, A. (1969). "Elastic storage of aquifers", In: Flow through porous media, (ed.) R.J.M. DeWiest, Academic Press, New York.